Dynamic Mode Decomposition Uncovers Hidden Oceanographic Features Around the Strait of Gibraltar

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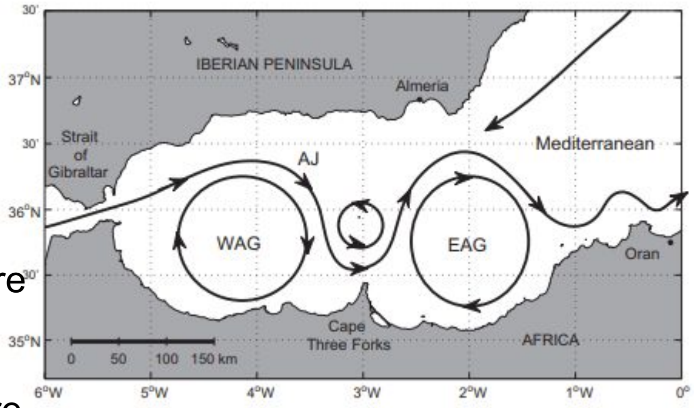
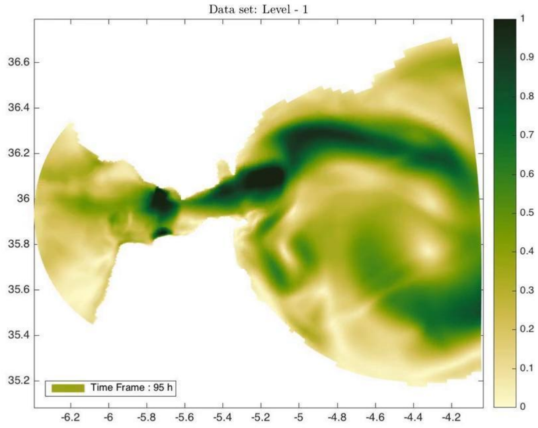
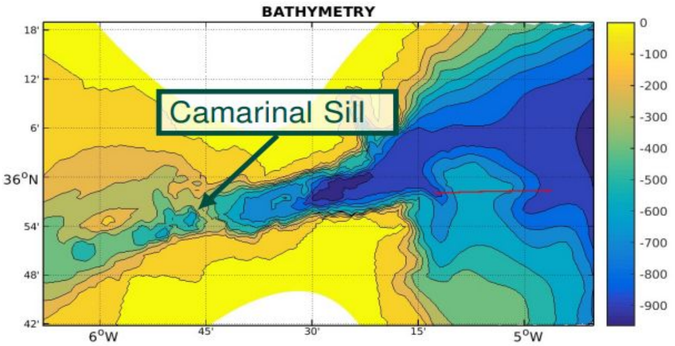
Strait of Gibraltar and Western Mediterranean

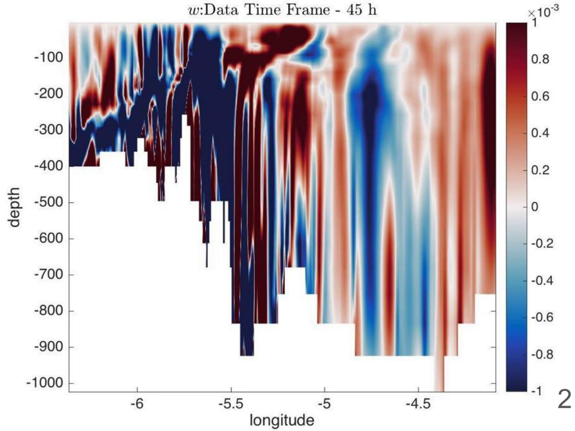
- AJ - Atlantic Jet
- WAG - Western Alboran Gyre
- EAG - Eastern Alboran Gyre
- Camarinal Sill (CS): Underwater elevation where an internal hydraulic jump gives rise to
    - internal waves comprising solitons
    - internal tides.

[Sánchez-Garrido, 2013]

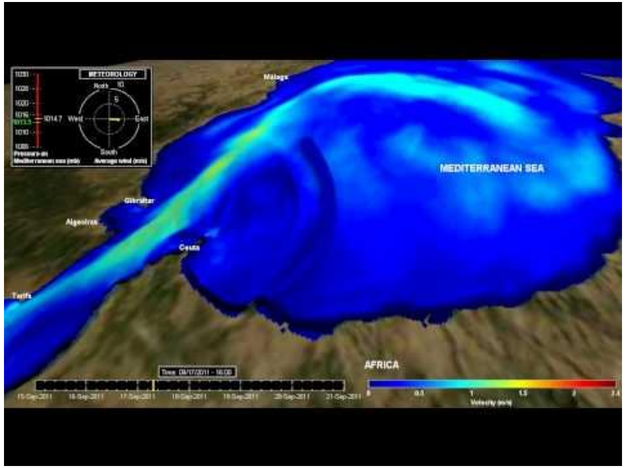


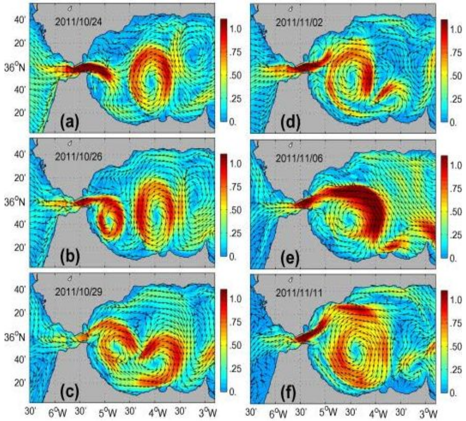






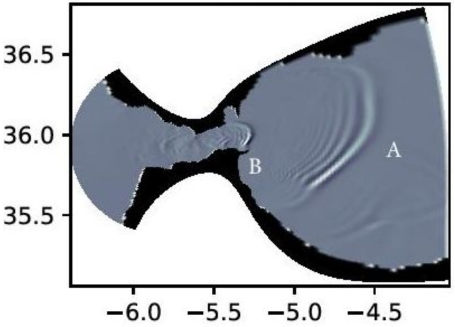
Known oceanographic features near Strait of Gibraltar





Western Alboran Gyre

Terra, 22 May, 11:30 UTC
MODIS, Enhanced True Colour RGB.
Source: NASA EOSDIS Worldview



Internal Waves

- Interested in recovering/decomposing known and hidden features
    - Gyres
    - Atlantic Jet
    - Internal waves
    - Hydraulic transitions
    - Special waves (such as Kelvin waves)
- Applied Dynamic Mode Decomposition (DMD) to simulated evolution

[Sánchez-Garrido, 2013], [Gofima UMA, YouTube], [NASA EOSDIS Worldview]

Numerical Model

MIT general circulation model (MITgcm; [8]) (3-D, finite-volume) simulating the 3D velocity components, temperature, salinity and density.

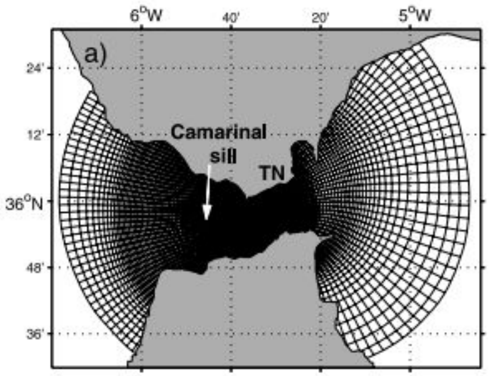
Input to analysis: velocity components at all grid points.

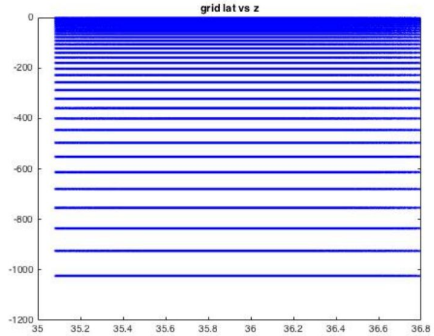
Grid:

- Nonuniform curvilinear orthogonal grid (190 x 96 x 32 nodes)
- Duration of 144h (6 days) with time step 1h

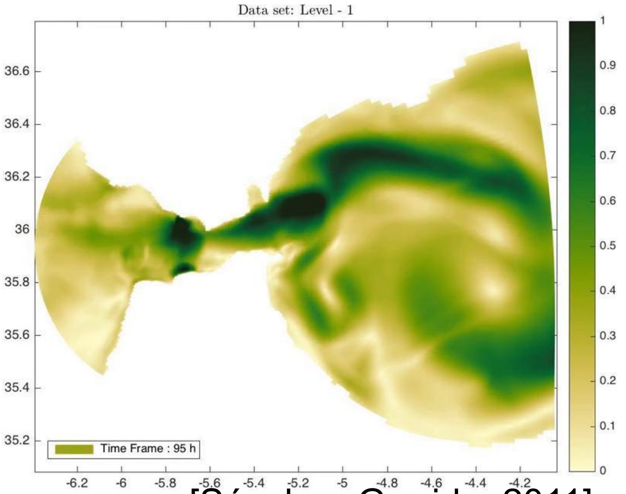
Forcing: Tidal currents extracted from intermediate resolution model and prescribed at open boundaries:

- **diurnal** (O1, K1) tides,
- **semidiurnal** (M2, S2) barotropic tides.





Simulated surface speed:



[Sánchez-Garrido, 2011]

Dynamic Mode Decomposition (DMD)

Why DMD?
- Isolate specific dynamic structures
- Equation free modeling
- Reduce the dimension of the data
- Simple algorithm and computation
- Can identify physically meaningful decomposition

Input Data

Let snapshot matrix $X = (S_i)$ where, S_i is column vector of state observable at step i

$$S_i := \begin{bmatrix} U_i \\ V_i \\ W_i \end{bmatrix}$$

$$X_1 = [\, S_1 \quad S_2 \ldots S_{N-1} \quad]$$
$$X_2 = [\, \quad S_2 \ldots S_{N-1} \quad S_N \,]$$

$$X_2 = \mathbb{K} X_1$$

where $\mathbb{K}$ is koopman operator

$$\mathbb{K}_t[g](x) = g \circ f_t(x)$$

Exact DMD

Input: $\mathbf{X_1}, \mathbf{X_2}$

1 $[U_r, \Sigma_r, V_r] = \mathrm{SVD}(\mathbf{X_1})$; `// Truncated SVD`

2 $\mathbf{K} = U_r^* \mathbf{X}_2 V_r \Sigma_r^{-1}$; `// Compression` $\mathbb{K} \downarrow \mathbf{K}$

3 $[\Psi, \Lambda] = \mathrm{eig}(\mathbf{K})$;`// diag(`$\Lambda$`) equals to eigenvalues of` $\mathbb{K}$

4 $\Phi = \mathbf{X_2} V_r \Sigma_r^{-1} \Psi$; `//` Φ `estimates eigenvectors of` $\mathbb{K}$

5 $\mathbf{b} = \Phi^\dagger \mathbf{X_1}[:, 1]$; `// Estimates the coefficients`

Output: $\Phi, \quad \Lambda, \quad \mathbf{b}$

$$X(t) = \begin{bmatrix} u(t) \\ v(t) \\ w(t) \end{bmatrix} \approx \sum_{i=1}^{r} b_i \Phi_i(x, y, z) e^{\omega_i t} \qquad \omega_i = \ln(\lambda_i)/\Delta t$$

[Tu, 2014], [Budišić, 2012]

Dynamic Mode Decomposition

Objective: Represent simulation data by a **linear, separation-of-variables data model**. Such linearization is **justified by Koopman operator theory**.

Spatiotemporal snapshot matrix

$$V(t,x) \approx \sum_{k=1}^{K} \Phi_k(x)\exp\{\Omega_k t\} b_k$$

Spatial profile

Time-evolution governed by eigenvalue

$$\Omega_k = \sigma_k + i\omega_k$$

Mode **magnitude**

DMD modes oscillate at a single (complex) time frequency.

Dominant modes: large mean L^2 norm

$$\tilde{b}_k := \frac{1}{T}\int_0^T \left|\exp\{\Omega_k t\} b_k\right|^2$$

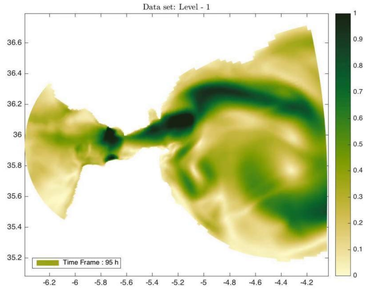
Real parts of eigenvalues imply growth/decay of a modes:

$$\operatorname{sign}\operatorname{Re}\Omega_k = \begin{cases} + & \text{Growth of mode} \\ - & \text{Decay of mode} \end{cases}$$

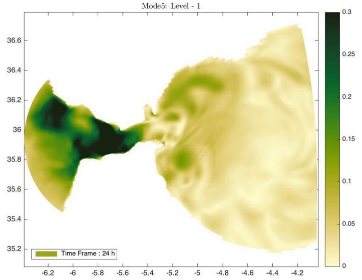
Imaginary parts of eigenvalues reported as oscillation period

$$P = 2\pi / \operatorname{Im}\Omega_k$$

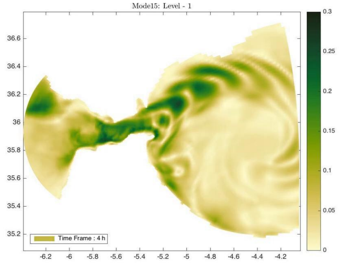
Input data:



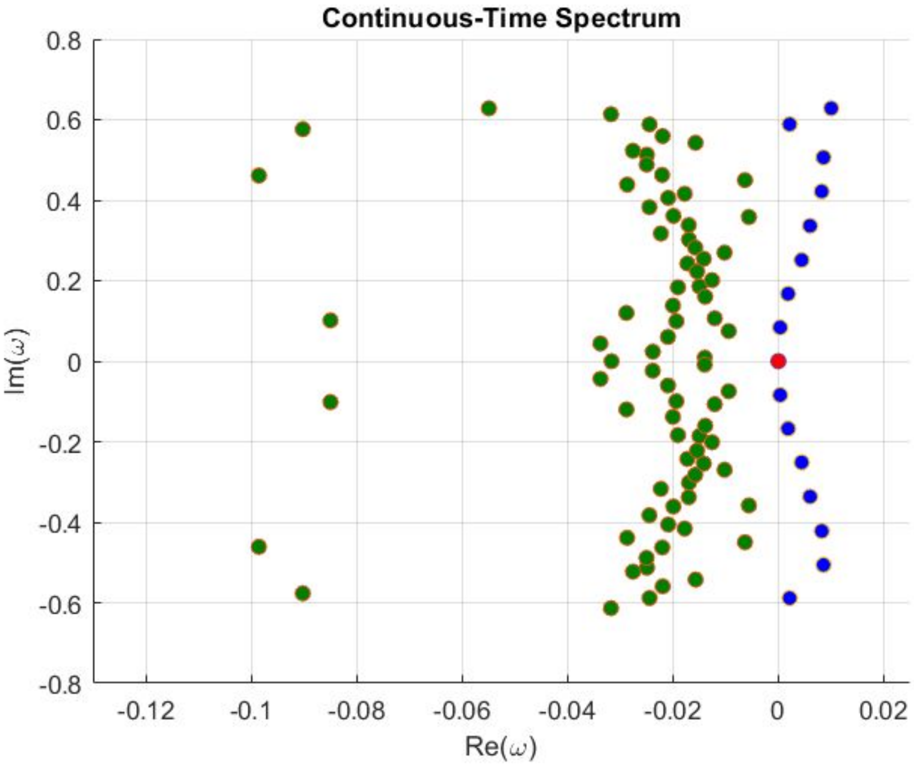
Semidiurnal dominant mode:



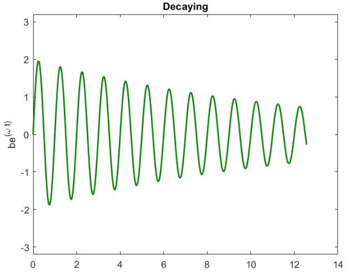
Diurnal dominant mode:



Behavior of Modes by Continuous- time Spectrum

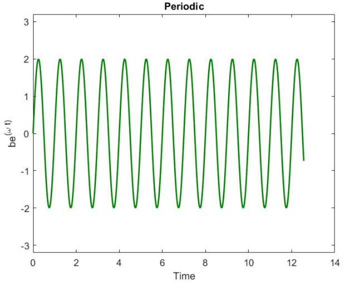


$Re(\omega) < 0$



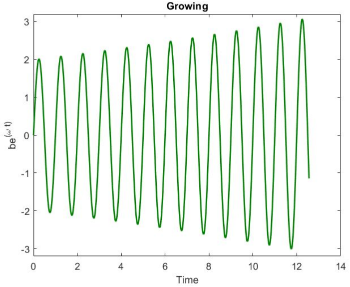
Decaying

$Re(\omega) = 0$



Periodic

$Re(\omega) > 0$

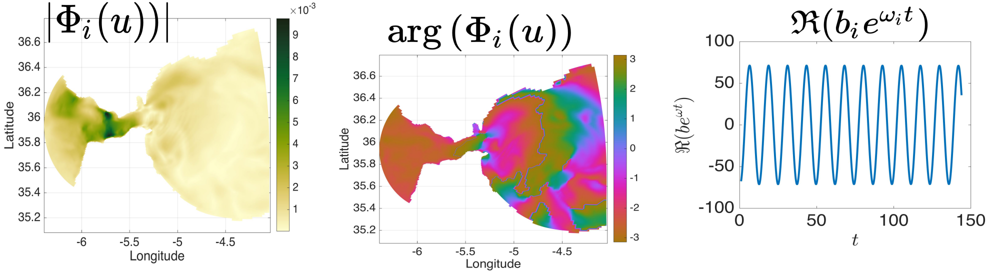


Growing

Best-Fit of Modes

$$X(t) = \begin{bmatrix} u(t) \\ v(t) \\ w(t) \end{bmatrix} \approx \sum_{i=1}^{r} b_i \Phi_i(x,y,z) e^{\omega_i t}$$

$$\text{Let } \omega_i = \sigma_i + i\rho_i$$



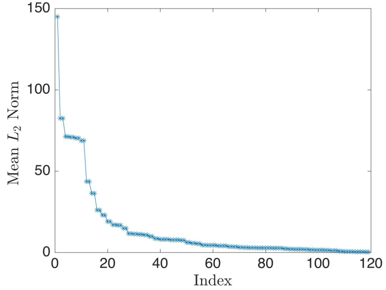
- DMD Algorithm decompose the data in to dynamical relevant flows features (same frequency).
- Noticed b, Φ, ω could be complex values.
- Eigenvalues ω describe the dynamics of the system.
- Magnitude and phase describes the how the spatial locations related each within a mode.

Mode Selection

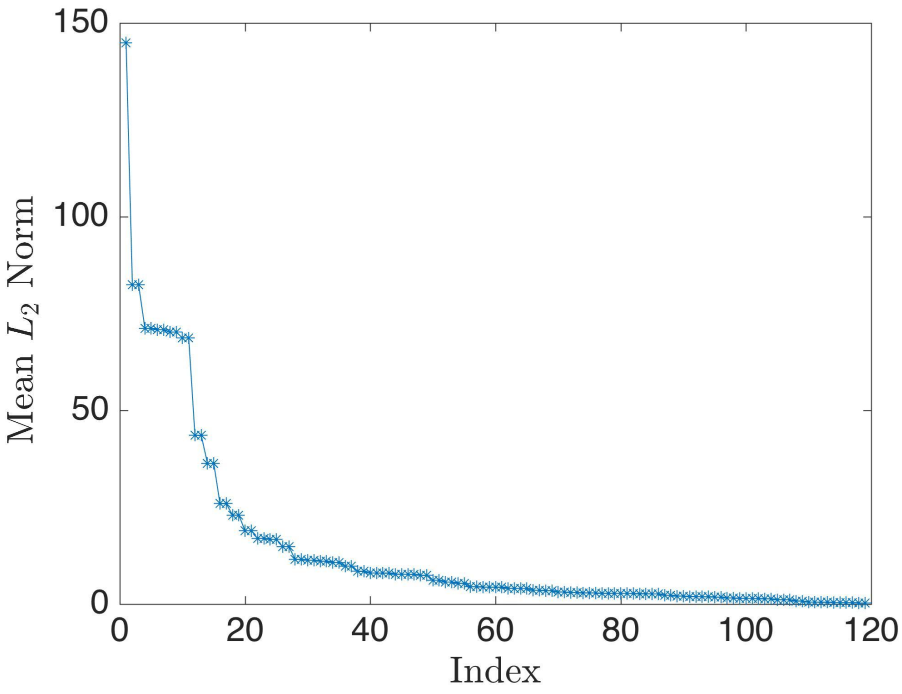
- L_2 norm contribution to the initial condition.
  - Sorting modes according to descending |b|.
- Average L_2-contribution across all snapshots

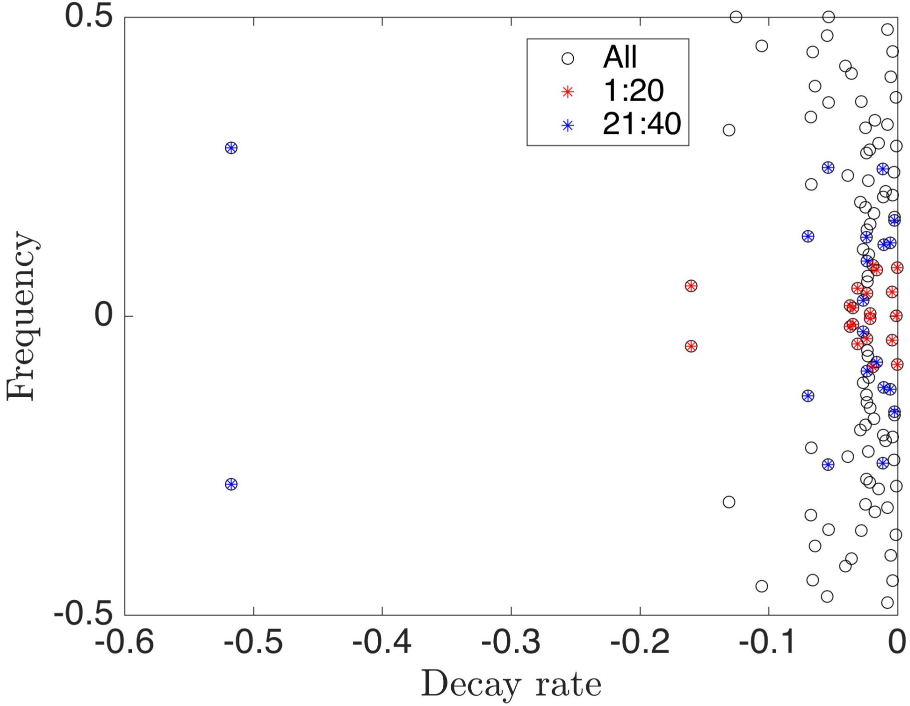
$$E_i = \sqrt{\frac{1}{T}\int_0^T |be^{\omega_i t}|^2 dt} = |b_i|\sqrt{\frac{e^{2\sigma_i T}-1}{2\sigma_i T}}$$

- Selection of modes based on the oscillation frequency associated with eigenvalues enables matching of modes to frequencies known a priori, e.g., forcing frequencies.



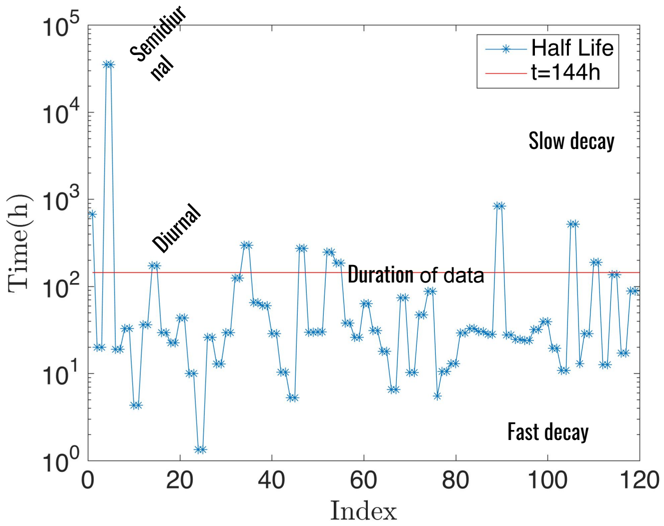
DMD for Gibraltar Dataset: Eigenvalue Spectrum





$$\omega_i = \ln(\lambda_i)/\Delta t$$

DMD Modes: Periods, decay rates, norms



Index	Half Life (h)	$P_i = \frac{2\pi}{\rho_i}$ (h)	Mean L2 Norm	|b|
1	668.10	Inf	144.84	155.63
3	19.91	73.47	82.44	259.25
5	35116.86	12.31	71.23	71.33
7	18.90	57.93	70.85	228.63
9	32.88	228.92	70.24	172.08
11	4.33	19.87	68.73	463.55
13	36.37	11.74	43.56	101.58
15	172.44	24.65	36.32	47.04
17	29.34	26.61	25.94	67.23
19	22.42	21.34	22.97	68.08
21	43.32	13.04	18.94	40.59
23	9.99	7.48	16.99	75.44
25	1.34	3.55	16.71	202.43
27	25.96	37.70	14.85	40.91
29	12.87	4.02	11.52	45.04
31	29.35	10.86	11.32	29.33
33	124.52	8.17	11.07	15.61
35	296.88	6.24	10.78	12.60
37	64.91	8.37	9.81	17.51
39	59.96	4.06	8.45	15.61
41	28.69	7.57	8.02	21.03
43	10.38	4.54	7.99	34.78
45	5.29	3.21	7.74	47.21
47	273.20	6.06	7.65	9.07
49	29.82	17.42	7.42	19.08

Background mode

Semidiurnal

Diurnal

DMD mode 1 is approximately non-oscillatory.

DMD modes 5 related to M2 tidal mode.

Period of DMD mode 15 is equals to M1 tidal period.

Tidal Component	Darwin	Period (hr)	Speed(°/hr)
Principal lunar semidiurnal	M_2	12.4206	28.9841
Principal solar semidiurnal	S_2	12	30
Smaller lunar elliptic diurnal	M_1	24.841	14.4921

$$\begin{bmatrix} u(t) \\ v(t) \\ w(t) \end{bmatrix} \approx \sum_{i=1}^{r} b_i \Phi_i(x,y,z) e^{\omega_i t}$$

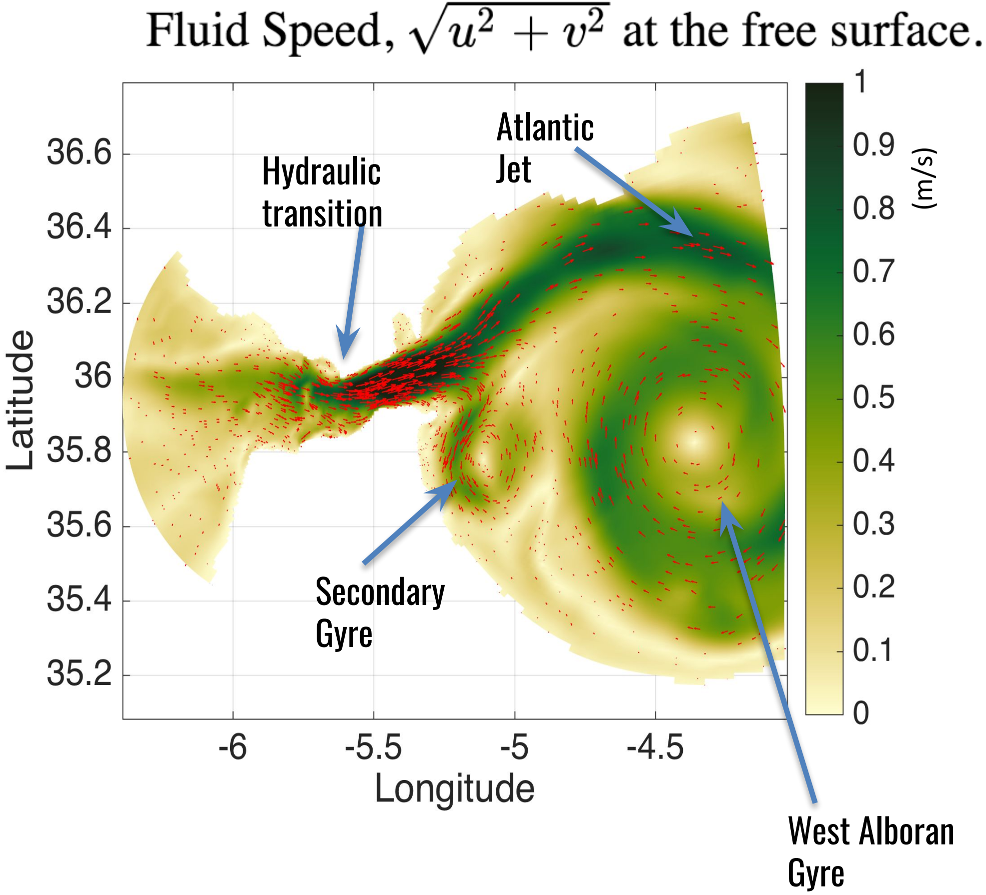
Eigenvalue: $\omega_i = \sigma_i + i\rho_i$

Half life Time(HLT): $\tau_i = \frac{-\ln(2)}{\sigma_i}$

Period: $P_i = \frac{2\pi}{\rho_i}$

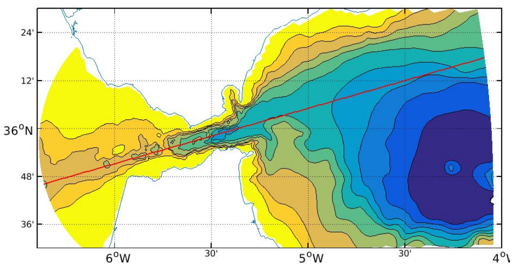
Mean L_2 Norm: $E_i = ||\Phi_i||^2 \frac{e^{2\sigma_i T} - 1}{2\sigma_i T}$

Mode 1: Non-oscillatory Background – Surface Speed



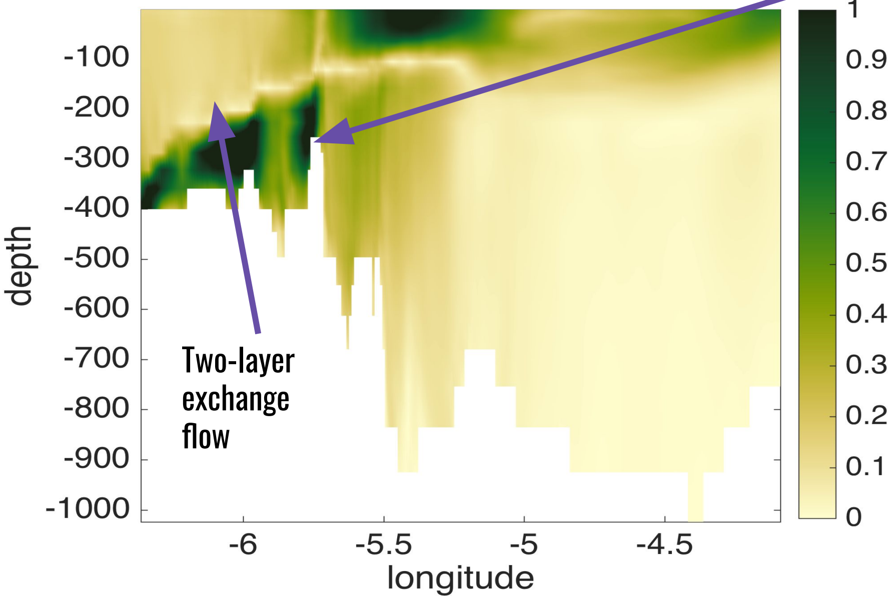
- Approximately non-oscillatory
- Persistent: Decay with half life time around 27 days (668 hours)
- Reveals several well-known features of the circulation in the region
    - West Alboran Gyre (WAG)
    - Atlantic Jet
    - the accelerated surface inflow in the Strait of Gibraltar.
- Also reveals a feature that is not as well known: secondary gyre that sits between the Ceuta and the WAG

Mode 1: Non-oscillatory Background – Vertical cross section

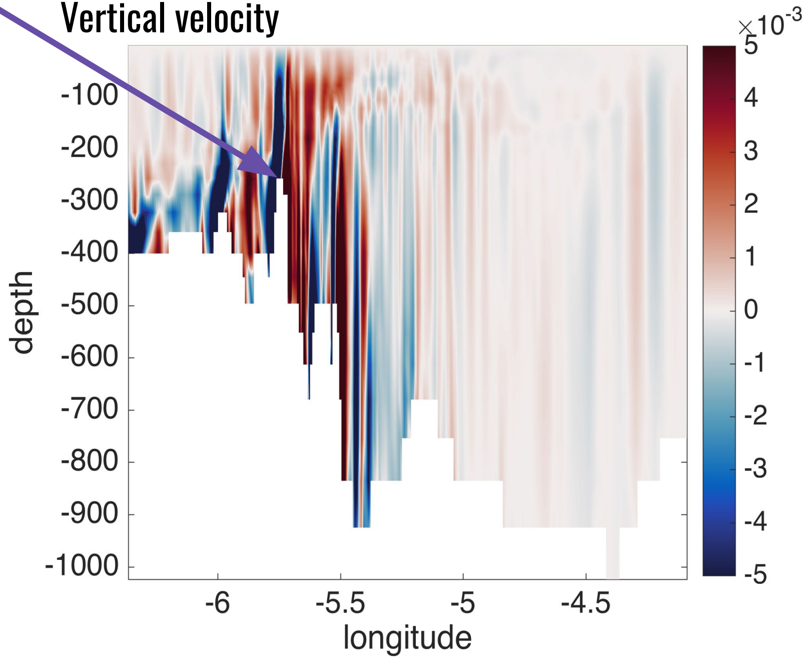


Hydraulic transition: westward flowing water spills over the Camarinal sill and gains speed

Horizontal speed

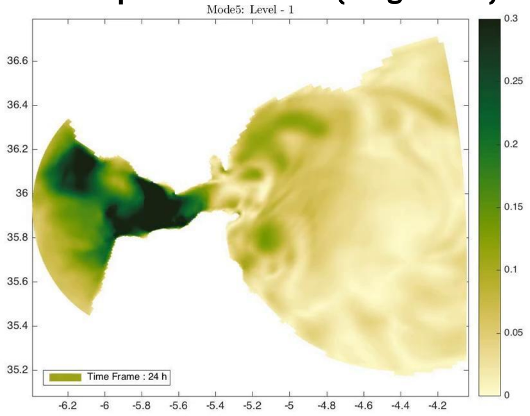


Vertical velocity

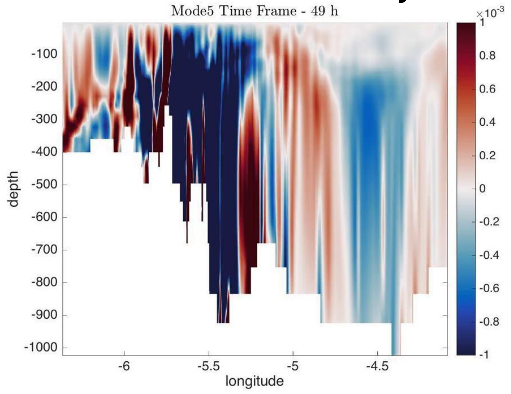


Mode 5: Dominant Semidiurnal (P=12.3 h)

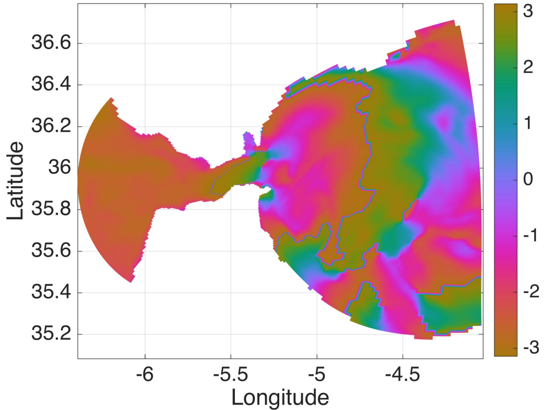
Horiz. speed at surface (magnitude)



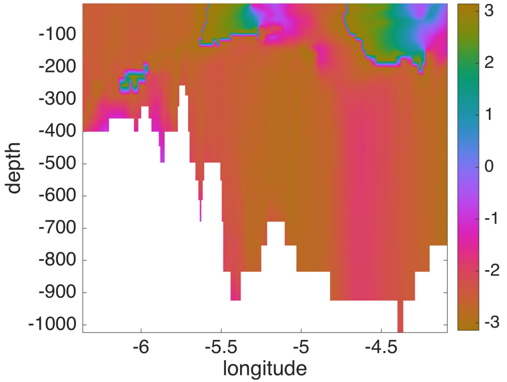
Cross-sec. vertical velocity



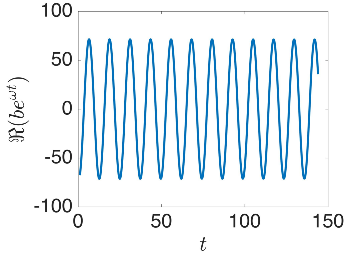
Horiz. speed at surface (phase)



Cross-sec. vertical velocity phase

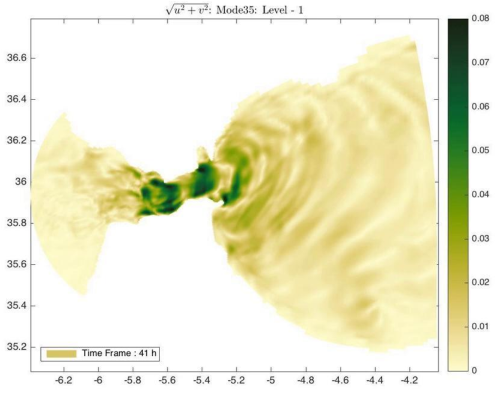


- Persistent: Effectively no decay
- Reveals internal waves radiating from the Strait into the Mediterranean.

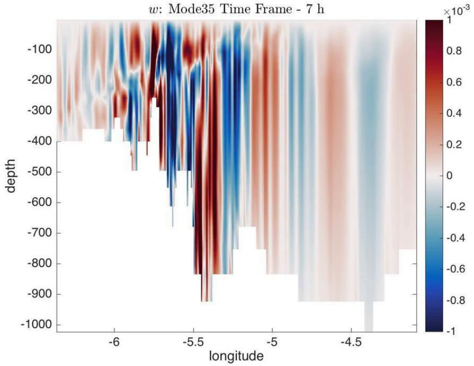


Mode 35: 2nd Harm. of Semidiurnal Tide (P=6.24h)

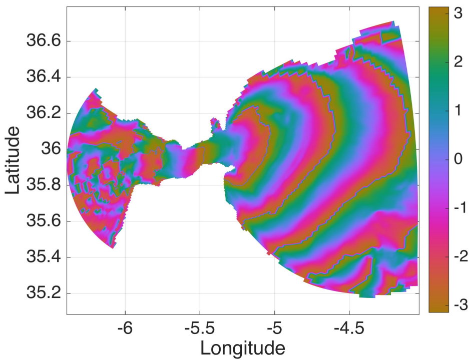
Surface horiz. speed magnitude



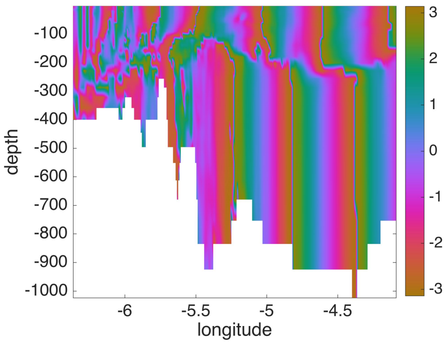
Cross-sec. vertical vel.



Surface horiz. speed phase



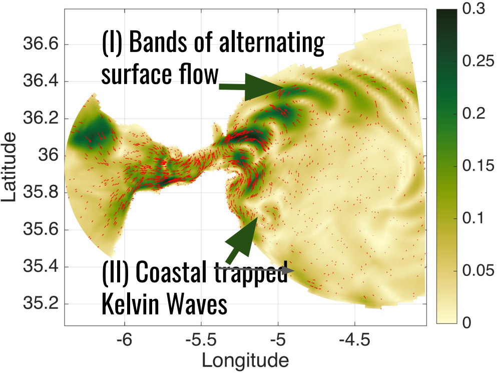
Cross-sec. vertical phase

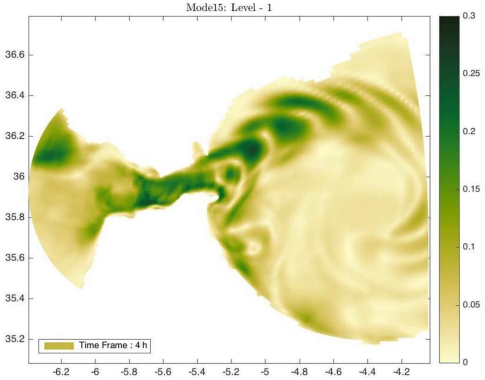


- Second harmonic of semidiurnal tide, and third harmonic of diurnal tides.
- Persistent: Decay with half life time around 297 hours (~2x available data length).
- Reveals internal waves radiating from the Strait into the Mediterranean.

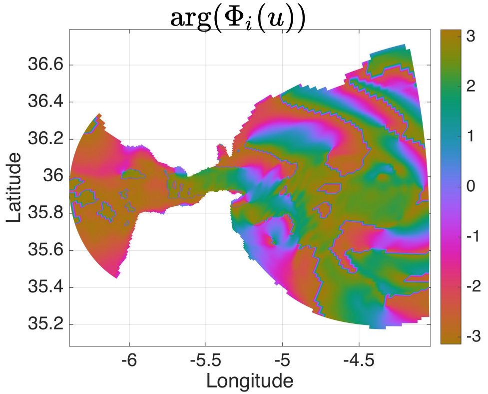
Mode 15: Dominant Diurnal (P=24.65h) – Surface Section

Surface horiz. speed magnitude





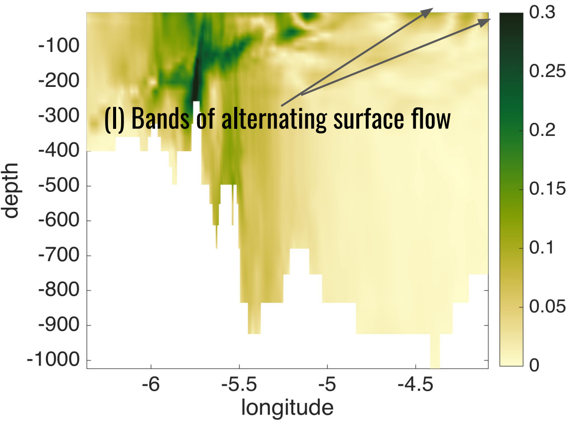
Surface horiz. speed phase

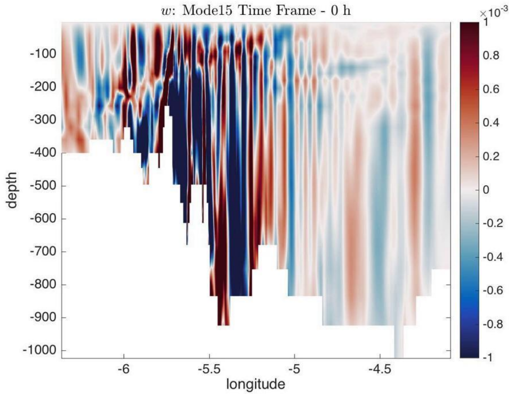


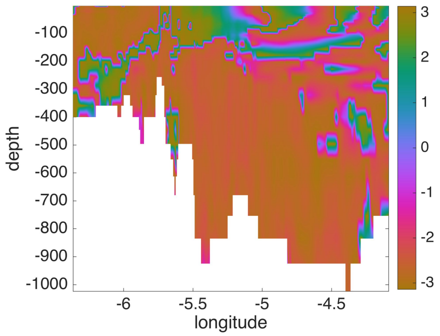
- Persistent: Decay half life around 172 h (7 days).
- (I) Isolates the meandering of the Atlantic Jet.
- Indicates that Atlantic Jet meanders are locked with a tide (first observation?).
- (II) Shows two patches of strong horizontal surface velocity along the southern coastline (Kelvin waves)

Hypothesis: Meanders originate from tidal pulses of vorticity generated in the strait and then carried by the jet.

Mode 15: Dominant Diurnal (P=24.65h) – Vertical Section



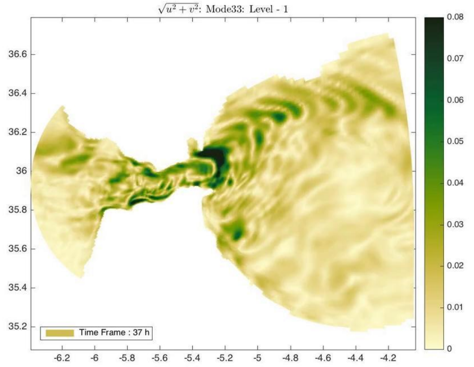




- Persistent: Decay half life around 172 h (7 days).
- (I) Isolates the meandering of the Atlantic Jet.
- Indicates that Atlantic Jet meanders are locked with a tide (first observation?).
- (II) Shows two patches of strong horizontal surface velocity along the southern coastline (Kelvin waves)

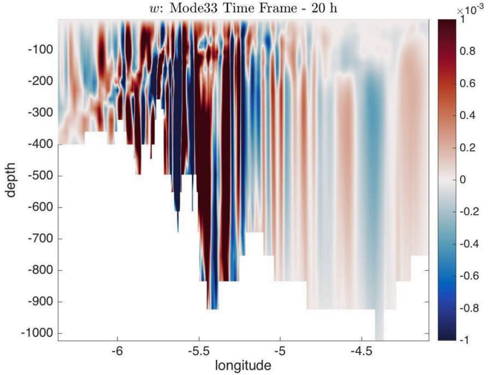
Mode 33: 3rd Harm. of Diurnal Tide (P=8.17h)

Horiz. speed at surface (magnitude)



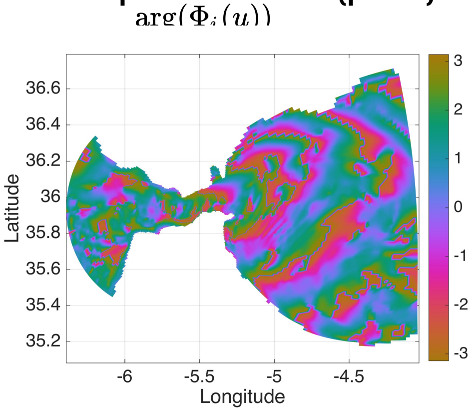
Meandering of the Atlantic Jet

Cross-sec. vertical velocity

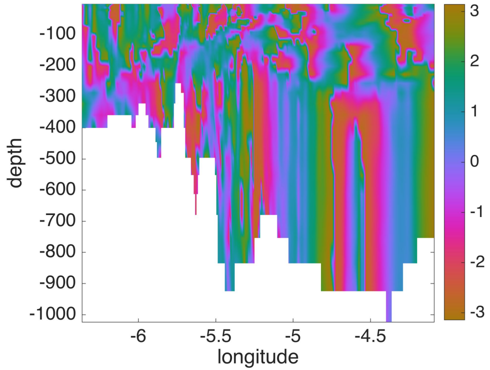


- Persistent: Decay with half life time around 124 hours.
- Exhibits internal wave activity and shows additional detail in Atlantic Jet meandering.

Horiz. speed at surface (phase)

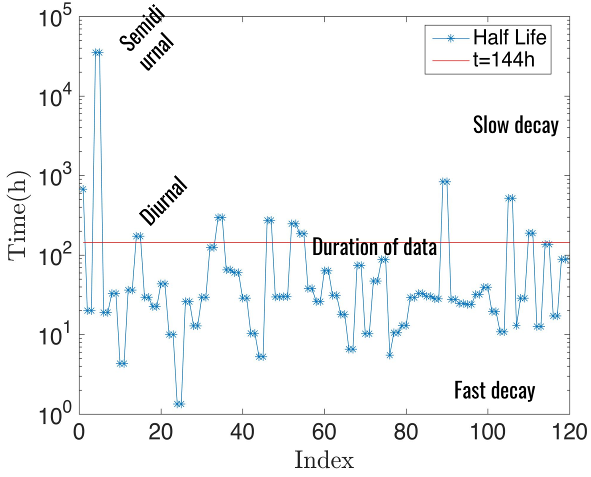


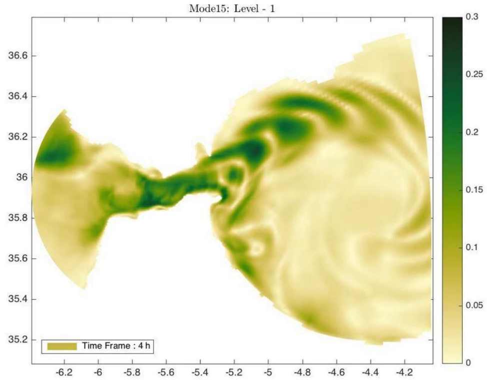
Cross-sec. vertical velocity phase



Summary

- Dynamic Mode Decomposition decomposes the data into modes that evolve according to a single (complex-valued) frequency.
- Clear time signature enables easy correlation with tidal components and identification of tidal harmonics.
- Individual DMD modes correlate with specific features and mechanisms of ocean dynamics: West Alboran Gyre, Atlantic Jet, hydraulic transition at Camarinal Sill, radiating internal waves, …
- DMD modes additionally isolate less-obvious features: secondary gyre at Ceuta, hydraulic jump, meandering of Atlantic Jet, Kelvin waves.





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