MA 132: Discussion 2

Wednesday, January 20, 2021 9:26 PM

office hours:
Tu : 11 – 1
Wed: 9–11 & 12–1

Agenda
- Integration of basic functions
- U-sub
- 6.1 Area between curves

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right)=x^n \quad \Rightarrow \int x^n\,dx=\frac{x^{n+1}}{n+1}+c,\ n\neq -1$$

$$\frac{d}{dx}(\ln|x|)=\frac{1}{x} \quad \Rightarrow \int \frac{1}{x}dx=\ln|x|+c$$

$$\frac{d}{dx}(e^x)=e^x \quad \Rightarrow \int e^x\,dx=e^x+c$$

$$\frac{d}{dx}(a^x)=(a^x\ln a) \quad \Rightarrow \int a^x\,dx=\frac{a^x}{\ln a}+c \quad (a>0)$$

$$\frac{d}{dx}(\sin x)=\cos x \quad \Rightarrow \int \cos x\,dx=\sin x+c$$

$$\frac{d}{dx}(\cos x)=-\sin x \quad \Rightarrow \int \sin x\,dx=-\cos x+c$$

$$\frac{d}{dx}(\tan x)=\sec^2 x \quad \Rightarrow \int \sec^2 x\,dx=\tan x+c$$

$$\frac{d}{dx}(\operatorname{cosec} x)=(-\cot x\operatorname{cosec} x) \Rightarrow \int \operatorname{cosec} x\cot x\,dx=-\operatorname{cosec} x+c$$

$$\frac{d}{dx}(\sec x)=\sec x\tan x \quad \Rightarrow \int \sec x\tan x\,dx=\sec x+c$$

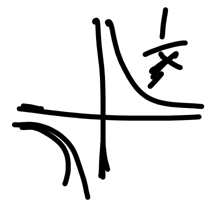
$$\frac{d}{dx}(\cot x)=-\operatorname{cosec}^2 x \quad \Rightarrow \int \operatorname{cosec}^2 x\,dx=-\cot x+c$$

Ex:

$$\int \cos x\,dx=\sin x+$$

$$\frac{d\,\boxed{\sin x}}{dx}\ ?=\cos x$$

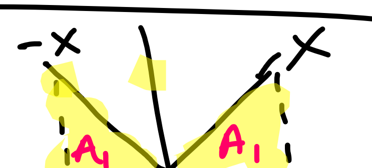
$$\int x^{-1}dx=\int \frac{1}{x}dx$$
$$=\ln|x|$$



$$\frac{d\arctan x}{dx}=\frac{1}{1+x^2} \Rightarrow \int\frac{1}{1+x^2}dx=\arctan x+C$$

$$\frac{d\arcsin x}{dx}=\frac{1}{\sqrt{1-x^2}} \Rightarrow \int\frac{1}{\sqrt{1-x^2}}dx=\arcsin x+C$$

$$\int^2 |x|\,dx=\int^0 -x\,dx+\int^2 x\,dx$$



$$= -\frac{x^2}{2}\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_{0}^{2}$$

$$= -\left(0 - \frac{(-2)^2}{2}\right) + \left(\frac{2^2}{2} - 0\right)$$

$$= +2 + 2$$

$$= 4$$

$$|x| = \begin{cases} -x & ; x \\ x & ; x \end{cases}$$

$$|-5| = -(-5) = 5$$

U-Sub

Recall: chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

$$\int f'(g(x))\, a \cdot g'(x)\, dx = \int f'(u)\, a \cdot du$$

choose $u = g(x)$ then $du = g'(x)\, dx$

Ex: ① $\int \frac{2x}{3x^2+1}\, dx = \int \frac{2}{u} \frac{du}{6}$

$$= \frac{2}{6} \int \frac{1}{u}\, du$$

$$= \frac{1}{3} \ln|u| + C$$

$$= \frac{1}{3} \ln|3x^2+1| + C$$

choose $u = 3x^2 + 1$

$$\frac{du}{dx} = 6x$$

$$\frac{du}{6} = \frac{6x\,dx}{6}$$

$$\frac{du}{6} = x\,dx$$

Ex: $\int \frac{x^3}{x^4 - 1}\, dx = \int \frac{1}{u} \frac{du}{4}$

choose $u = x^4 - 1$

Find

$\frac{du}{dx} = 4x^3$

$du = 4x^3 dx$

$\frac{du}{4} = x^3 dx$

$= \frac{1}{4} \int \frac{1}{u} du$

$= \frac{1}{4} \ln|u| + C$

$= \frac{1}{4} \ln|x^4 - 1| + C$

$$\int e^{2x} dx = \int e^u \frac{du}{2}$$

$u = 2x$

$\frac{du}{dx} = 2$

$du = 2dx$

$\frac{du}{2} = dx$

$= \frac{1}{2} \int e^u du$

$= \frac{1}{2} e^u + C$

$= \frac{1}{2} e^{2x} + C$

$\frac{d}{dx}\left[\boxed{\frac{e^{2x}}{\cancel{2}}}\right] =$

Ex: $$\int e^{3x} dx = \frac{e^{3x}}{3} + C$$

$$\int \cos(\pi x) dx = \frac{\sin(\pi x)}{\pi} + C$$

$$\int \sin(3x) dx = -\frac{\cos(3x)}{3} + C$$

$$\int e^{-x} dx = \frac{e^{-x}}{-1} + C$$

$$\int \sin\left(\frac{x}{2}\right) dx = \frac{-\cos\left(\frac{x}{2}\right)}{\left(\frac{1}{2}\right)} + C$$

$\int (a f(x) + b g(x)) dx$

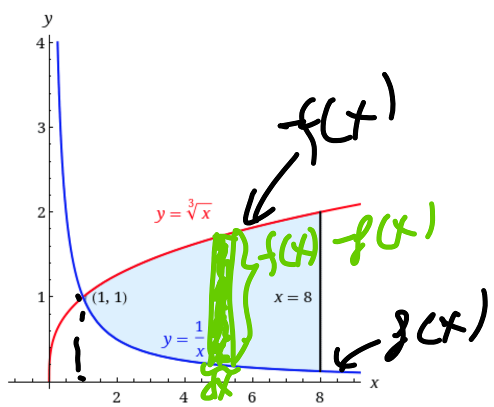
$$= a\int f(x)\,dx + b\int g(x)\,dx$$

Ex: $\int 3x^2+2\,dx = 3\int x^2\,dx + 2\int 1\,d$

$$= 3\,\frac{x^3}{3} + 2x + C$$

6.1 Area between curves

Find the area of the shaded region.



$$\int_1^8 f(x) - g(x)\,dx$$

$$= \int_1^8 \sqrt[3]{x} - \frac{1}{x}\,dx$$

$$= \int_1^8 x^{1/3} - \frac{1}{x}\,dx$$

$$= \int_1^8 x^{1/3}\,dx - \int \frac{1}{x}\,dx$$

$$= \left.\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right|_1^8 - \ln|x|\Big|_1^8$$

$$= \left.\frac{x^{4/3}}{4/3}\right|_1^8 - \ln|x|\Big|_1^8$$

$$= \frac{3}{4}\left(8^{4/3} - 1\right) - (\ln 8 - \cancel{\ln 1})\ \rightarrow 0$$

$$= \frac{45}{4} - \ln 8$$

$$\frac{1}{3}+1 = \frac{1}{3} + \frac{3}{3} = \frac{4}{3}$$

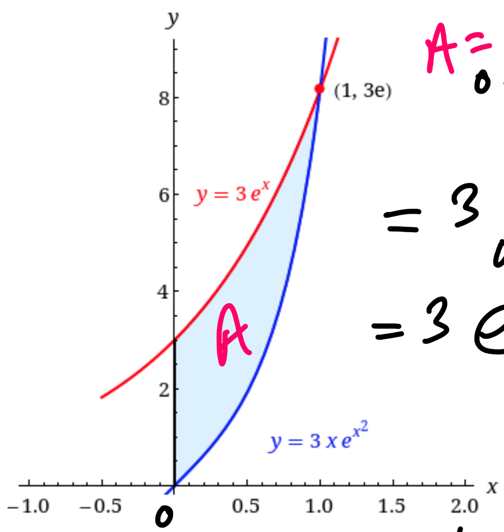
$$1 \div \frac{4}{3}$$

$$1 \times \frac{3}{4}$$

$$\ln 1 = 0$$

$$e^{\boxed{0}} = 1$$

$(2^{\cdot})^{\cdot} = 2^{\cdot} =$



$A = \int_0^1 3e^x - 3xe^{x^2}\,dx$

$= 3\int_0^1 e^x\,dx - 3\int_0^1 xe^{x^2}\,dx$

$= 3e^x\Big|_0^1 - 3\cdot\frac{1}{2}\cdot e^{x^2}\Big|_0^1$

$= 3(e^1 - e^0) - \frac{3}{2}\cdot(e^1 - e^0)$

$\int x e^{x^2}\,dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

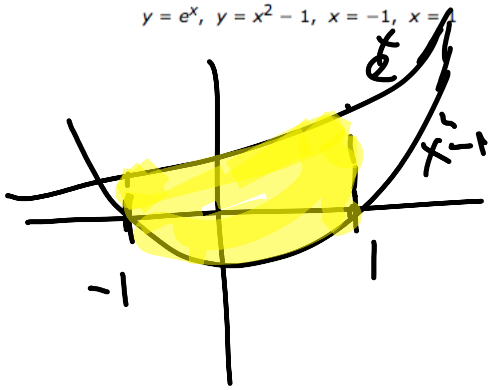
$= \int e^u \frac{du}{2}$

$= \frac{1}{2}\int e^u\,du$

$= \frac{1}{2}e^{x^2}$

Sketch the region enclosed by the given curves and Find the area of the region.

$y = e^x,\ y = x^2 - 1,\ x = -1,\ x = 1$

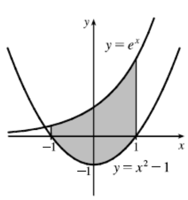


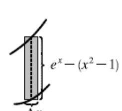
$\int_{-1}^{1} e^x - (x^2 - 1)\,dx$

Solution or Explanation

$A = \int_{-1}^{1}[e^x - (x^2 - 1)]\,dx = \left[e^x - \frac{1}{3}x^3 + x\right]_{-1}^{1}$

$= \left(e - \frac{1}{3} + 1\right) - \left(e^{-1} + \frac{1}{3} - 1\right) = e - \frac{1}{e} + \frac{4}{3}$





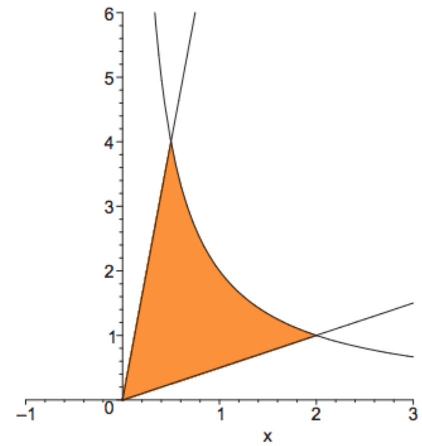
$y = 2/x, \quad y = 8x, \quad y = \frac{1}{2}x, \quad x > 0$

$2/x = 8x \quad \Leftrightarrow \quad 0.25 = x^2 \quad \Leftrightarrow$

$x = \pm 0.5$ and $2/x = \frac{1}{2}x \quad \Leftrightarrow$

$4 = x^2 \quad \Leftrightarrow \quad x = \pm 2$, so for $x > 0$,

$$\begin{aligned} A &= \int_0^{0.5} \left(8x - \frac{1}{2}x\right) dx + \int_{0.5}^{2} \left(\frac{2}{x} - \frac{1}{2}x\right) dx \\ &= \int_0^{0.5} \left(\frac{15}{2}x\right) dx + \int_{0.5}^{2} \left(\frac{2}{x} - \frac{1}{2}x\right) dx \\ &= \left[\tfrac{15}{4}x^2\right]_0^{0.5} + \left[2\ln(|x|) - \tfrac{1}{4}x^2\right]_{0.5}^{2} \\ &= \tfrac{3.75}{4} + \left[\left(2\ln(2) - \tfrac{4}{4}\right) - \left(2\ln(0.5) - \tfrac{0.25}{4}\right)\right] \\ &= 2\ln(4) \end{aligned}$$



$y = 3\cos(\pi x), \quad y = 12x^2 - 3$

By inspection, the curves intersect at $x = \pm\frac{1}{2}$.

$$\begin{aligned} A &= \int_{-1/2}^{1/2} [9\cos(\pi x) - (8x^2 - 2)]dx \\ &= 2\int_0^{1/2} (9\cos(\pi x) - 8x^2 + 2)dx \quad \text{[by symmetry]} \\ &= 2\left[\frac{9}{\pi}\sin(\pi x) - \frac{8}{3}x^3 + 2x\right]_0^{1/2} = 2\left[\left(\frac{9}{\pi} - \frac{1}{3} + 1\right) - 0\right] \\ &= 2\left(\frac{9}{\pi} + \frac{2}{3}\right) = \frac{18}{\pi} + \frac{4}{3} \end{aligned}$$